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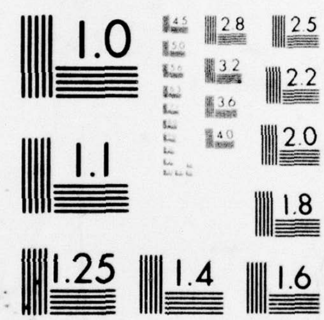


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# ADAPTIVE APPROXIMATION TECHNIQUES IN OPTIMIZATION

FINAL REPORT


E. Polak


October 1976

U. S. ARMY RESEARCH OFFICE

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This report describes our results in devising algorithms for solving three types of optimization problems: (1), nonlinear programming problems with both equality and inequality constraints, (2), nonlinear programming problems with functional constraints of the form $\max_{\omega\in\Omega} \phi(x,\omega)\leq 0$, as well as ordinary equality and inequality constraints (these arise in engineering design), and (3), constrainted optimal control problems. Our algorithms are presented in an implementable form which specifies all the important approximations.

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Introduction

The bulk of the work carried out under this contract was concerned with algorithms for solving three types of optimization problems: (1) nonlinear programming problems with both equality and inequality constraints, (2) nonlinear programming problems with functional constraints of the form $\max_{\omega\in\Omega} \phi(x,\omega)\leq 0$, as well as ordinary equality and inequality constraints (these arise in engineering design), and (3) constrainted optimal control problems.

The thing that all these problems have in common is that algorithms for their solution, in "conceptual" form, require an infinite number of "elementary" operations per iteration and hence must be converted into "implementable" form by the introduction of approximations. Typically, the approximations are applied to the solutions of a differential equation, or to the solution of a system of equations and inequalities, or to the computation of the constrained maximum of a function. Now, it is not difficult to see that arbitrary approximation schemes may lead to implementable algorithms that do not converge. Furthermore, the invention of approximation schemes consistent with convergence is by no means easy. Finally, in many engineering applications, even approximate solutions are quite costly and hence computational efficiency of the resulting implementable algorithm is a very important consideration in the evaluation of an approximation scheme.

During the three years of this project, (i) we have contributed substantially to the theory of adaptive approximations as it applies to the construction of implementable algorithms. In addition, (ii) We

have made use of our theory in developing several new implementable algorithms for the classes of problems mentioned earlier. We have developed techniques for enlarging the region of convergence without loss of rate, of such algorithms as Newton's method or secant methods, by grafting them onto slower, but more robust algorithms. We have developed a new theory of convergence in the sense of relaxed controls for optimal control algorithms and, finally, we have explored a few problems in multicriteria optimization.

## Major Results

### 1) Adaptive Approximation Theory

For the purpose of illustration, consider the following relatively simple problem, which is the absolutely simplest example of the type of problem occurring in computer aided design.

$$\min_{x\in\mathbb{R}^n} f(x) \tag{1}$$

with

$$f(x) = \max\{\phi(x,\omega)\,|\,\omega\in\Omega\} \tag{2}$$

and $\phi:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^1$ differentiable, $\Omega\subset\mathbb{R}^m$ compact. To simplify matters further, suppose that for each $x\in\mathbb{R}^n$ there is only one $\omega\in\Omega, \omega(x)$, such that $f(x)=\phi(x,\omega)$. Then $f(\cdot)$ is differentiable and (1) can be solved by Armijo's gradient method: ($\beta\in(0,1)$, $\alpha\in(0,1)$ parameters)

$$x_{i+1}=x_i-\beta^{k_i}\nabla\phi(x_i,\omega(x_i)) \tag{3}$$

where $k_i \geq 0$ is the smallest integer satisfying

$$f(x_i - \beta^{k_i} \nabla\phi(x_i, \omega(x_i))) - f(x_i) \leq -\beta^{k_i} \alpha \| \nabla\phi(x_i, \omega(x_i)) \|^2 \tag{4}$$

Quite obviously the conceptual version (3)-(4) is naive, because it requires $k_i$ evaluations of $f(x)$ at each iteration, and even a fair approximation to $f(x)$ and $\omega(x)$ can be quite expensive. Furthermore, when $\phi(x_i)$ is not convex, there is no certain way to evaluate $f(x)$ at all! We have developed a theory for modifying algorithms such as (3)-(4) into efficient implementable forms with demonstrable convergence properties and we have developed a number of new algorithms by making systematic use of our theory. These algorithms are in the gradient projection, reduced gradient, and min max algorithm families. A key feature of our theory is that it permits the use of very coarse approximations in the early iterations and that it uses a test to determine when accuracy should be increased. In particular, we wish to draw attention to [10,16,17], which describe three algorithms that perform very well in computer aided design applications. These algorithms solve problems of the form $\min\{f^0(x) \mid g^j(x) \leq 0,\ j=1-m;\ \max_{\omega \in \Omega} f^i(x,\omega) \leq 0,\ i = 1-\ell\}$. The results dealing with gradient projection and reduced gradient methods are reported in [3,5]. Our numerical experiments show that algorithms with adaptive approximation features are many times faster than similar algorithms which use constant precision throughout.

2) <u>Exact Penalty and Multiplier Methods</u>

As a representative problem, consider

$$\min\{f^0(x) \mid f^1(x) \leq 0\} \tag{5}$$

with $f^0, f^1: \mathbb{R}^n \to \mathbb{R}^1$ differentiable. This problem has the same solutions as the unconstrained exact penalty problem

$$\min_{x \in \mathbb{R}^n} \ (f^0(x) + k|f^1(x)|) \tag{6}$$

provided the penalty $k>0$ is sufficiently large. An algorithm for solving (5) via (6) must incorporate a scheme for computing a satisfactory $k$ which is finite and it must be able to cope with the nondifferentiability of $|f^1(x)|$. In [7] we have described a general scheme for automatically computing a parameter for an algorithm which is to be inserted into the algorithm as a test with logic. Applications of this scheme to new optimal control and nonlinear programming algorithms are reported in [11,12,13,14]. We also wish to draw attention to the novel way in which the use of the penalty functions $|f^1(x)|$ has been eliminated in [11] by the addition of inequality constraints and in [14] by more sophisticated techniques.

3) <u>Stabilization of Locally Convergent Algorithms</u>

Consider the problem

$$\min_{x \in \mathbb{R}^n} \ f(x) \tag{6}$$

with $f: \mathbb{R}^n \to \mathbb{R}^1$ three times continuously differentiable. The simplest version of Newton's method:

$$x_{i+1} = x_i - H(x_i)^{-1} \nabla f(x_i) \tag{7}$$

(with $H(x) = \partial^2 f(x)/\partial x^2$) converges only if $x_0$ is sufficiently close to a solution. For the case $H(x)>0$ for all $x \in \mathbb{R}^n$, Goldstein proposed the modified method

$$x_{i+1}=x_i-\lambda_i H(x_i)^{-1}\nabla f(x_i) \tag{8}$$

with $\lambda_i=\beta^{k_i}$ ($\beta\in(0,1)$) satisfying a certain line search rule. The method (8) has a greater region of convergence than (7), but it breaks down when $H(x)>0\ \forall x$ does not hold. We have developed algorithms which use a test to switch between formulas such as (8) and another one, such as, e.g.,

$$x_{i+1}=x_i-\lambda_i\nabla f(x_i) \tag{9}$$

in such a way that the region of convergence is enlarged over (8) and quadratic rate of convergence is still retained. Details can be found in [1,2,4].

4) Convergence Theory for Optimal Control

An optimal control algorithm constructs an infinite sequence of controls $\{u_i\}$ which are measurable functions from an interval $[t_o,t_f]\to\mathbb{R}^m$. Generally, the sequence $\{u_i\}$ cannot be shown to have limit points in $L_\infty^m[t_o,t_f]$ or in $L_2^m[t_o,t_f]$ or in $L_1^m[t_o,t_f]$ which are the usual spaces for analysis. However, it is possible to place this sequence in correspondence with a sequence of relaxed controls $\{\hat{u}_i\}$ which always has limit points in the topology of relaxed controls. We have developed a convergence theory for optimal control algorithms, which, by means of relaxed controls, enables us to make meaningful statements about their convergence. Specifically, although ordinary limits for the sequence of controls do not exist, the corresponding trajectories do have $L_\infty$ limits and we show that these limit trajectories satisfy certain optimality conditions which emanate from the relaxed Pontrygin Maximum Principle. The theory is described in [9], applications can be found in [14,15].

5) Multicriteria Optimization

So far, we have made only very modest progress in this area. Our results are described in [6,8]. Our continuing aim is to construct computational aids which facilitate the decision process on the Pareto-optimal (i.e. trade-off) surface.

## Conclusion

The last three years have been productive years for us during which we have obtained several significant results.

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